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TESTING THE TRANSITION LAYER MODEL OF QUASI-PERIODIC OSCILLATIONS IN NEUTRON STAR X-RAY BINARIES

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ABSTRACT

We compare the theoretical predictions of the transition layer model with some observational features of quasi-periodic oscillations (QPOs) in neutron star X-ray binaries. We found that the correlation between horizontal branch oscillation (HBO) frequencies and kilohertz (kHz) QPO frequencies, the difference between the low-frequency QPOs in atoll sources and HBOs in Z sources, and the correlation between the frequencies of low-frequency QPOs and break frequencies can be well explained by the transition layer model, provided the neutron star mass is around 1.4 solar mass and the angle between magnetosphere equator and accretion disk plane is around 6 degree. The observed decrease of peak separation between two kHz QPO frequencies with the increase of kHz QPO frequencies and the increase of QPO frequencies with the increase of inferred mass accretion rate are also consistent with the theoretical predictions of transition layer model. In addition, we derive a simple equation that can be adopted to estimate the angle (δ) between magnetosphere equator and accretion disk plane by use of the simultaneously observed QPO frequency data. We estimate these angles, in the range of 4 to 8 degrees, for five Z sources and two atoll sources. The nearly constant δ value for each source, derived from the different sets of simultaneously observed QPO frequency data, provides a strong test of the theoretical model. Finally, we suggest that the similar transition layer oscillations may be also responsible for the observed QPOs in accretion-powered millisecond X-ray pulsar and Galactic black hole candidates.

Subject headings: accretion, accretion disks — stars: neutron — stars: binaries: general — X-Rays: stars

1. INTRODUCTION

Quasi-periodic Oscillations (QPOs) have been found to be a very common feature of accreting systems around compact objects. In neutron star low-mass X-ray binaries (LMXBs) with low magnetic field, at least four types of QPOs, namely the 15-60 Hz horizontal branch oscillations (HBOs; van der Klis et al. 1985), the 6-20 Hz normal branch oscillations (NBOs; Middledich & Priedhorsky 1986) and the 300-1200 Hz lower and upper kilohertz (kHz) QPOs (van der Klis et al. 1996), have been discovered by several X-ray satellites including *EXOSAT, GINGA*, and the *Rossi X-Ray Timing Explorer (RXTE).* In spite of the different evolution tracks in the X-ray color-color diagram (Hasinger & van der Klis 1989), the low-frequency QPOs in atoll sources seem to have the similar feature to those of the HBOs in Z sources (Strohmayer et al. 1996, Wijnands & van der Klis 1997). With the advantage of *RXTE*, 20 sources have now shown kHz QPOs and 18 of them have shown two simultaneous kHz peaks (see van der Klis 2000 for a review). The peak separations of two kHz QPOs have been observed to decrease considerably in five sources when the kHz QPO frequencies increase (van der Klis *et al.* 1997; Méndez et al. 1998a,b; Ford et al. 1998, Méndez & van der Klis 1999, Markwardt et al. 1999). Other sources may also show similar variations (Psaltis et al. 1998). It has been found that the HBO frequencies of Z sources and the frequencies of low-frequency QPOs in atoll sources are well correlated with the kHz QPO frequencies (Psaltis et al. 1999a). Similar correlation of low and high frequency QPOs might also exist for some other neutron star systems and Galactic black hole candidates. In addition, tight correlations of the HBO frequencies of Z sources and of the frequencies of low-frequency QPOs in atoll sources and black hole candidates with the break frequencies shown in their power density spectra have also been found recently (Wijnands & van der Klis 1999). These two correlations strongly imply that similar mechanisms may be responsible for the break frequency, the low-frequency QPOs and the high-frequency QPOs in both neutron star and black hole X-ray binaries.

There are several QPO models that are currently in debate. Due to the difficulty in producing kHz QPOs, the previous magnetospheric beat-frequency model proposed by Alpar & Shaham (1985) has been modified as sonic-point beat-frequency model (Miller, Lamb & Psaltis 1998). In this model the upper kHz QPOs arise from the clumps moving with the Keplerian frequency at the sonic point that is near the innermost stable circular orbit of neutron star. The lower kHz QPO frequency is thus the beat frequency between the upper kHz QPO frequency and the neutron star spin frequency. However, this model, if without further modifications, predicts a constant peak separation between two kHz QPO frequencies, which is contrary to the observations. Moreover, recent investigation on the boundary layer accretion onto neutron star indicated that the accretion flow is always subsonic and there is probably no such a sonic point at all near the neutron star surface (Popham & Sunyaev 2001). This leads to further doubt about the sonic-point beat-frequency model. Another attractive model is the relativistic precession model(RPM) proposed by Stella & Vietri (1998, 1999), who identified the upper kHz QPO frequency with that of an Keplerian orbit in the disk and the lower kHz QPO frequency and HBO frequency with, respectively, the periastron precession frequency and twice of the nodal precession frequency of this orbit. This model can qualitatively explain the observed correlations between these three QPO frequencies (Stella, Vietri & Morskink 1999), but the precise match requires some fine tuning of additional free parameters such as the orbital eccentricity and the periastron distance. Moreover, in order to explain the HBOs and its correlation with kHz QPOs, this model requires that the I/M value (where I is the momentum of inertial and M is the mass of neutron star) be 2 times larger than that predicted by realistic neutron star equation of state (Psaltis et al. 1999b). In addition, RPM model requires the neutron star mass in the range of 1.9-2.2$M_\odot$ in order to explain the correlation between low and high QPO frequencies, which is significantly larger than the measured value, 1.1$M_\odot$<M<1.6$M_\odot$, for neutron stars in radio pulsar binaries (Thorsett & Chakrabarty 1999; Finn 1994). The problem to make a neutron star with mass larger than 2.2$M_\odot$ arises because it would require accretion of material of at least 0.8$M_\odot$, which means that these X-ray binaries would have to be rather old and would have to have spun up rapidly for 90% of their lifetime (Lai, Lovelace & Wasserman 1999). The same problem probably exists for the new model proposed by Psaltis & Norman (2000), who derived the similar characteristic QPO frequencies as in RPM in the inner accretion disk.

Another alternative model for QPOs in neutron star X-ray binaries is the disk transition layer model (hearafter TLM) proposed recently by Titarchuk & Osherovich (1999), Osherovich & Titarchuk (1999a,b) and Titarchuk, Oshervich & Kuznetsov (1999). The geometry of the transition layer model has been clearly shown in Figure 1 of Titarchuk *et al.* (1999). In this "two-oscillator" model, the lower kHz QPO frequency is identified with the Keplerian frequency in a viscous transition layer between Keplerian disk and neutron star surface. Assuming the magnetic field of neutron star is low (in the range of 10^7 to 10^9 *Gauss*), the Alfvén radius, depending mainly on the magnetic field and accretion rate, is at most of several radii of neutron star. The size of the magnetosphere, as well as that of the transition layer, is thus very small. Viscous oscillations in this transition layer produce the observed noise break and a low frequency QPO. In addition, the adjustment of the Keplerian disk to the sub-Keplerian layer may create conditions favorable for the formation of a hot blob in the transition layer. This blob, when thrown out into the rotating magnetosphere from the transition layer, participates in the radial oscillations with Keplerian frequency. Under the influence of the Coriolis force, such a blob, assumed to be a Keplerian oscillator, oscillates both radially and perpendicular to the disk. This produces two harmonics of another low-frequency QPO (HBO in Z sources) and the upper kHz QPO. These six different frequencies have been identified in two atoll sources (4U 1728-24 and 4U 1702-42) and Z source Sco X-1 (Titarchuk et al. 1999; Osherovich & Titarchuk 1999a,b). The observed correlations of QPO frequencies for these sources seem to be consistent with the theory of TLM.

In this paper, we derive more theoretical predictions for TLM and compare them with the available observational data of QPOs in neutron star X-ray binaries. The consistency between model predictions and observational data without assuming larger neutron star mass and a stiff equation of state suggests that TLM is perhaps a more competitive model than others. We also suggest that a similar model may be also applied to the cases of millisecond X-ray pulsars and Galactic black hole candidates.

2. CHARACTERISTIC QPO FREQUENCIES IN THE TRANSITION LAYER MODEL

As indicated by Titarchuk & Osherovich (1999) and Osherovich & Titarchuk (1999b), there are six characteristic QPO frequencies in the TLM. First, the fundamental frequency is assumed to be the Keplerian frequency, namely,

$$\nu_k = \frac{1}{2\pi}(\frac{GM}{R^3})^{1/2}, \tag{1}$$

where G is the gravitational constant, M is the mass of neutron star and R is the radius of an orbit in the transition layer. The linear Keplerian oscillator with frequency $\Omega_k/2\pi$ in the frame of reference rotating with rotational frequency $\Omega/2\pi$ (not perpendicular to the disk plane) is known to have an exact solution describing two branches of oscillations (see §39 in Landau & lifshitz 1960). Assuming Ω is constant, the dispersion relation of the frequency of oscillation ω can be derived as:

$$\omega^4 - (\Omega_k^2 + 4\Omega^2)\omega^2 + 4\Omega^2\Omega_k^2 sin^2\delta = 0, \tag{2}$$

where δ is the angle between Ω and the normal to the Keplerian oscillation. If δ is assumed to be small, one can obtain simple expressions of two characteristic oscillation frequencies from the dispersion relation. The radial eigenmode has a frequency

$$\nu_h = \sqrt{\nu_k^2 + (\frac{\Omega}{\pi})^2}, \tag{3}$$

where Ω is the angular velocity of the rotating magnetosphere in the case of neutron star X-ray binaries. If the magnetosphere corotates with the neutron star, we have $\Omega = \Omega_0$ where Ω_0 is the rotating angular velocity of neutron star. The frequency ν_h is the analog of the hybrid frequency in plasma physics (Akhiezer *et al.* 1975; Benson 1977). On the other hand, the frequency of the vertical eigenmode is

$$\nu_L = \frac{\Omega}{\pi}\frac{\nu_k}{\nu_h} sin\delta, \tag{4}$$

These three characteristic frequencies, ν_k, ν_h and ν_L, are assumed to account for the frequencies of lower kHz QPOs, upper kHz QPOs and HBOs in Z sources, respectively. Because the second harmonics of HBO are often seen in Z sources, they are interpreted as $2\nu_L$, counting as the other characteristic frequency.

From equations (3) and (4), we can derive several very useful relations among these characteristic frequencies. First, the correlation between the HBO frequency and two kHz QPO frequencies can be expressed as:

$$\nu_L = \nu_k\sqrt{1 - (\frac{\nu_k}{\nu_h})^2} sin\delta. \tag{5}$$

Second, the difference between two kHz QPO frequencies can be related to ν_k as

$$\Delta\nu = \nu_h - \nu_k = \nu_k(\sqrt{1+(\frac{\Omega}{\pi\nu_k})^2} - 1). \qquad (6)$$

Third, if the Keplerian oscillation is aligned with the accretion disk plane, the angle between magnetosphere equator and disk plane can be determined by

$$\delta = arcsin(\frac{\nu_h\nu_L}{\nu_k\sqrt{\nu_h^2-\nu_k^2}}). \qquad (7)$$

Because three frequencies in the right of equation (7) are all observational quantities, this equation perhaps suggests a useful method to determine δ for neutron star X-ray binaries.

The other two characteristic frequencies in the TLM are related to the two typical timescales in the transition layer. One is the radial drift timescale of matter through the layer bounded between disk and neutron star, namely, $t_r \sim (R_{out} - R_0)/v_r$, where R_0 is the radius of neutron star (taken to be $6GM/c^2$ in this paper) and v_r is the radial drift velocity. This timescale gives the characteristic viscous frequency as

$$\nu_v \simeq \frac{\gamma\nu}{(R_{out}-R_0)R} \simeq 2\pi\alpha\gamma\nu_k(\frac{H}{R})^2\frac{r_{out}}{r_{out}-1}, \qquad (8)$$

where r_{out} is defined as $r_{out} = R_{out}/R_0$, and γ is the Reynolds number given by $\gamma = \dot{M}/4\pi\rho\nu H = v_rR/\nu$ where $\dot{M}$ is the mass accretion rate and ρ is the mass density. We adopt the standard viscosity prescription $\nu = \alpha c_s H = \alpha\Omega_k H^2$, where c_s is the local sound speed and H is the scale height (Shakura & Sunyaev 1973). The relation between γ and r_{out} can be obtained by considering the radial transport of angular momentum and the boundary conditions at R_0 and R_{out} (Titarchuk & Osherovich 1999). Assuming that at R_0, $\Omega = \Omega_0$ and at R_{out}, $\Omega = \Omega_k$ and $d\Omega/dr = d\Omega_k/dr$ and defining $A = \Omega_{k0}/\Omega_0$ (where Ω_{k0} is the Keplerian angular velocity at R_0), r_{out} can be determined by following equation:

$$(\frac{3}{2}-\gamma)Ar_{out}^{2-\gamma} + (\gamma-2)r_{out}^{-\gamma} + \frac{1}{2}A = 0. \qquad (9)$$

Another characteristic timescale in the transition layer is the diffusion time scale given by $t_{diff} \sim (R_{out} - R_0)^2/(l_dv_r)$, where l_d is the diffusion length scale in the transition layer. It gives the break frequency as

$$\nu_{break} \simeq \frac{l_d}{(R_{out}-R_0)}\nu_v = \frac{l_d}{R_0}\frac{\nu_v}{r_{out}-1}. \qquad (10)$$

This expression of break frequency is similar as that given in equation (29) of Psaltis & Norman (2000) if l_d is replaced by R. Similar to the argument given by Psaltis & Norman (2000), the reason that ν_{break} observed as a frequency break may be that the transition layer acts as a filter band so that the response is constant if the oscillation frequency is lower than the inverse diffusion time scale and the response decreases above it. Titarchuk & Osherovich (1999) described l_d as the mean free path of a particle, however, this introduced another uncertain quantity in the TLM theory. In this paper, using a similar approach adopted in the standard accretion disk model (Shakura & Sunyaev 1973), we assume that l_d is smaller than the vertical and radial size of the transition layer and take l_d/R_0 as a constant less than unity for simplicity (in §3.3 we will show this assumption is consistent with the observed data of break frequency). The equations (8), (9) and (10) can be used to calculate the correlations between other low-frequency QPO (besides HBO) frequency, break frequency and Keplerian frequency. The original descriptions of viscous and break frequencies shown in equations (12) and (13) in Titarchuk & Osherovich (1999) have been improved here (see equations (8) and (10)) by considering a more explicit expression of the radial drift timescale.

3. COMPARISONS OF MODEL PREDICTIONS WITH THE OBSERVATIONAL DATA

There are a lot of observations made by several X-ray satellites on QPOs in neutron star X-ray binaries. But only until very recently the correlations among the frequencies of low and high-frequency QPOs, the variation of kHz QPO frequencies and the difference between HBOs and other low-frequency QPOs have been systematically studied with plenty of QPO data. In this section we try to compare the predictions from the TLM, as indicated by equations in the section above, with the observational results.

3.1. *Correlation between HBOs and kilohertz QPOs in Z sources*

Recently, Psaltis *et al.* (1999a) have compiled the low and high-frequency QPO data of Z sources and atoll sources, as well as black hole candidates. They found a tight correlation between HBO frequency and the frequency of lower kHz QPO. The data points for the Z sources are consistent with the empirical relation $\nu_{HBO} \sim (42 \pm 3\mathrm{Hz})(\nu_1/(500\mathrm{Hz}))^{0.95\pm0.16}$when the frequency of lower kHz QPO ν_1 less than 550 Hz. When ν_1 larger than 500 Hz, ν_{HBO} increases slowly with ν_1. This feature seems to be difficult to fit by the RPM.

In Figure1, we plot ν_{HBO} and ν_1 data for five Z sources and compare it with the predictions from both RPM and TLM. The fit parameters for RPM ($M = 1.95M_\odot$, $a/M = 0.22$) are taken from Stella *et al.* (1999) and a is the spin parameter of the neutron star. It is evident that RPM predicts a steeper correlation than the observed one and can not explain the flatness of such a correlation when $\nu_1 > 500$ Hz. Any fine tuning of the fit parameters M and a/M can not provide a better fit. However, when we plot the theoretical curves of TLM according to the equation (5) by choosing quite reasonable parameters $\nu_h - \nu_k = 300$ Hz and δ around 6^o, the match of model prediction and observation data is quite well. In addition, we also plot the observed correlation between ν_{HBO} and upper kHz frequency ν_2 for five Z sources in Figure 2. In this case, the RPM can not fit the observation data if we still choose the parameter $M = 1.95M_\odot$, $a/M = 0.22$. Even when we choose the fit parameter $M = 1.95M_\odot$, $a/M = 0.30$, the fit is still not satisfactory. Therefore, our comparison shows that RPM, at least in its current version, is difficult to fit the correlations between simultaneously observed HBO frequencies and kHz QPO frequencies. On the contrary,

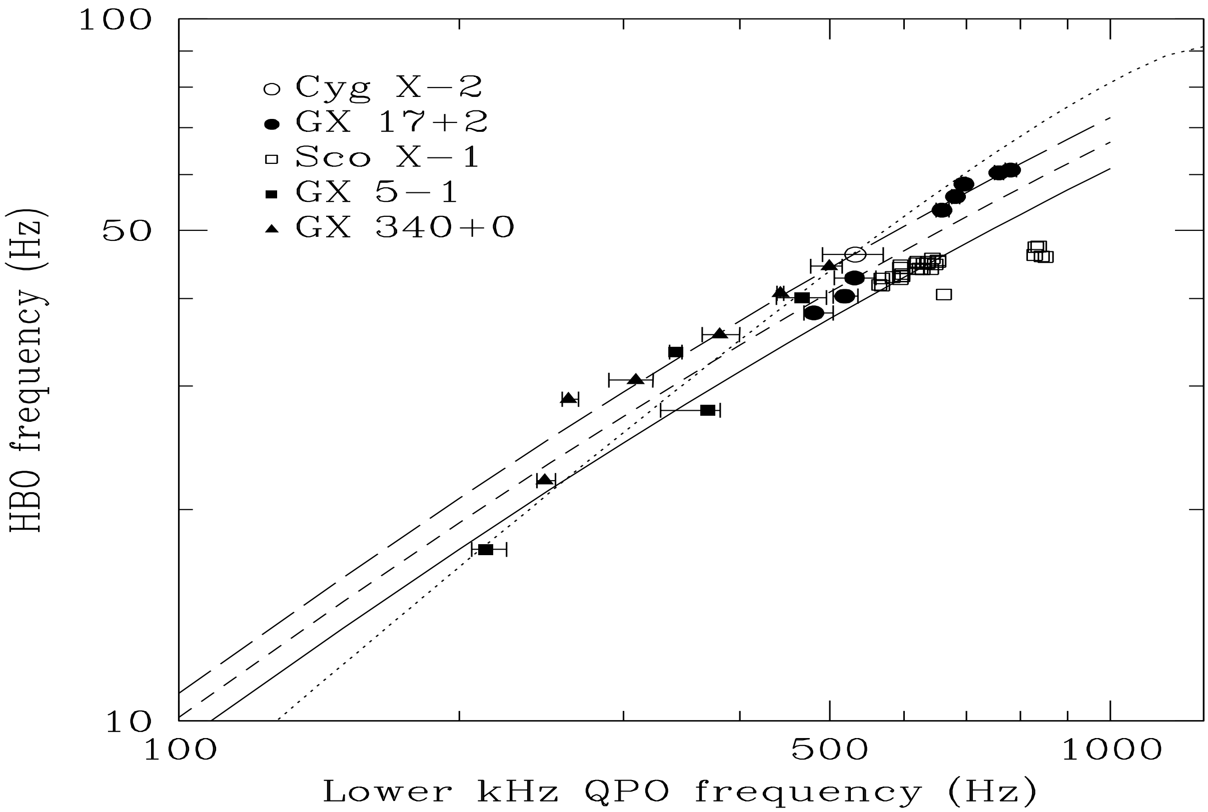

FIG. 1.— The correlation between HBO frequencies and lower kHz QPO frequencies of Z sources. The solid, short-dashed and long-dashed lines represent TLM predictions with parameters $\Delta\nu$ = 300 Hz and $\delta = 5.5^o$, 6^o and 6.5^o, respectively. The dotted line represents the prediction of RPM with $M = 1.95M_\odot$ and $a/M = 0.22$. The error bars, if not shown, are comparable to the size of the symbols.

from Figure 1 and Figure 2 we can see clearly that TLM predicts rather flatter slopes than RPM. With the same parameters, TLM can provide good fits to both the ν_{HBO} – ν_1 and $\nu_{HBO} - \nu_2$ correlations.

For some sources (especially GX17+2 and GX 5-1) in Figure1, it seems that the slope is steeper than that predicted by TLM and perhaps matches better with the RPM prediction. However, we must note that the TLM predictions plotted in figures 1 and 2 were based on a very simple assumption ($\nu_h - \nu_k = 300$Hz). In fact, the peak separation between two kHz QPO frequencies, $\nu_h - \nu_k$, is certainly not constant for individual sources (see §3.4). If we take the observed values of ν_h and ν_k directly and use equation (5) to predict ν_L, the agreement with the observations will be more close for individual sources plotted in figures 1 and 2. On the other hand, with reasonable parameters, we noted that RPM is unable to fit both the observed correlations shown in figures 1 and 2 consistently. Even if the RPM prediction could better match the slopes of GX17+2 and GX 5-1 in Figure 1, it fails to match the observed slopes for these sources in Figure 2 if the same parameters were used.

3.2. *Difference between low frequency QPOs in atoll sources and HBOs in Z sources*

Although there are some reports on the similar QPOs in atoll sources to HBOs in Z sources (Strohmayer *et al.* 1996, Wijnands & van der Klis 1997), it can be clearly seen from Figure 2 in Psaltis *et al.* (1999a) that the frequencies of these low-frequency QPOs (10-50 Hz) of atoll sources have steeper dependence on the lower kHz QPO frequency than those of HBOs in Z sources. In Figure 3, we plot this correlation and compare it with the empirical relation found for HBOs in Z sources. Obviously the difference between these low-frequency QPOs and HBOs is large. It is therefore quite possible that these low-frequency QPOs may have different origin from HBOs in Z sources. Actually, the similar low-frequency QPOs (usually seen as extra noise components) have been found simultaneously with HBOs in Z source Sco X-1 and GX 17+2 (van der Klis *et al.* 1997; Titarchuk *et al.* 1999; Wijnands & van der Klis 1999), while the similar HBO-like frequencies have been identified in two atoll sources 4U 1728-34 and 4U 1702-42 (Titarchuk *et al.* 1999; Osherovich & Titarchuk 1999b).

In Figure 3, it can be clearly seen that these low-frequency QPOs in Z source Sco X-1 follow the same correlation with ν_1 as other atoll sources. This probably suggests that the low-frequency QPOs in both Z and atoll sources have the same physics origin and they are obviously different from HBOs. In TLM, the frequency of this

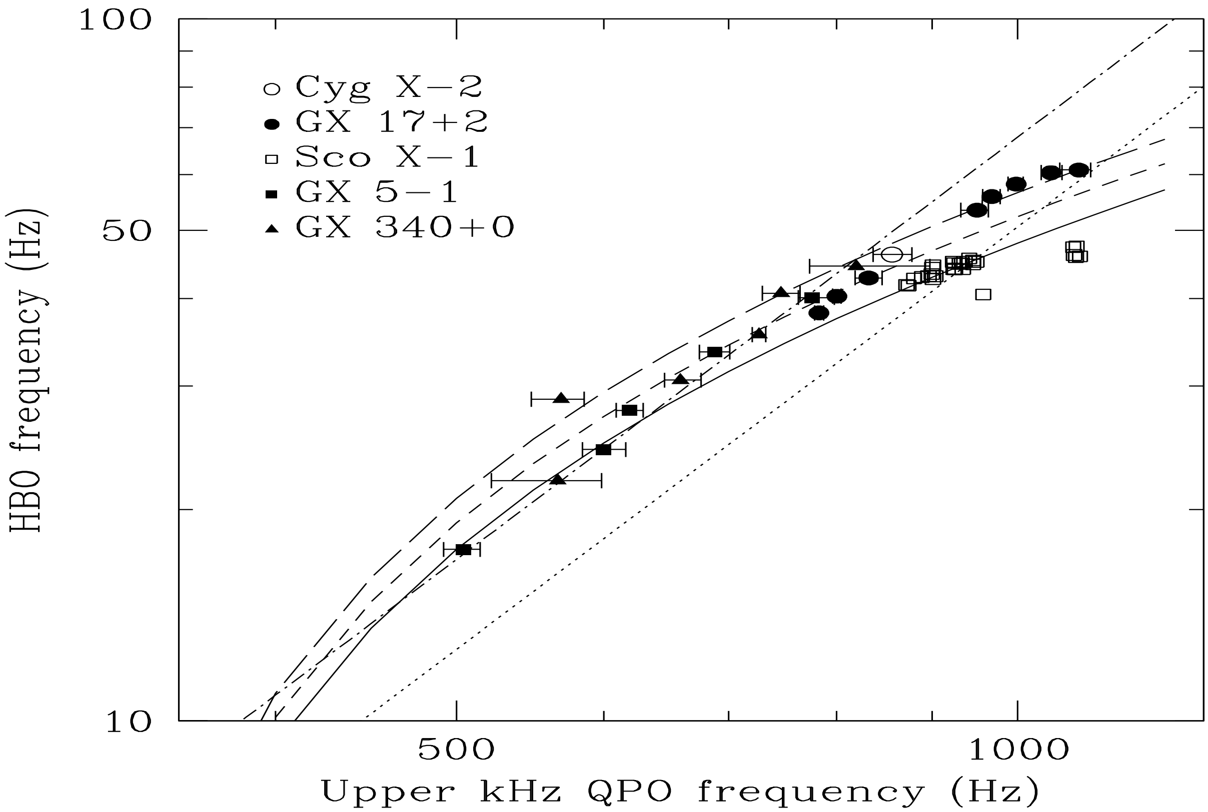

FIG. 2.— The correlation between HBO frequencies and upper kHz QPO frequencies of Z sources. The lines have the same meanings as in Figure1. The dot-dashed line represents the prediction of RPM with $M = 1.95M_{\odot}$ and $a/M = 0.30$.

kind of QPOs is described as the viscous frequency in the transition layer, therefore the mechanism to produce the low-frequency QPOs is clearly different from HBOs. Using equations (8) and (9) and assuming $\alpha(H/R)^2 = 7.0\times10^{-4}$, M around $1.4M_{\odot}$ and r_{out} in the range of 1.2 to 1.8 (namely γ in the range of 6 to 20), we plot the model prediction of TLM in Figure 3 (the result is insensitive to the spin frequency of neutron star, $\nu_0 = 300$ Hz is assumed to make the plot). We can see the close agreement between theory and observations. For KS 1731-26 and 4U 1735-44, the data points are a little away from others. They are likely associated with the NBOs of Sco X-1 (Psaltis *et al.* 1999a), though they can be still fit by assuming a smaller value of $\alpha(H/R)^2$. However, we noted that for 4U 1735-44, recent observations indicated the existence of a 67Hz QPO together with a possible 900Hz lower kHz QPO (Wijnands *et al.* 1998c). If we plot this point in Figure 3, it will also locate in the range of TLM prediction.

In Figure 3, only the data of February 16, 1996 for 4U 1728-34 are plotted because in these observations the low kHz QPOs can be clearly distinguished from the simultaneously observed upper kHz QPOs (Ford & van der Klis 1998). The agreement of the observation data with our predictions assuming M around $1.4M_{\odot}$ suggests that the compact object in 4U 1728-34 may not be a strange star as argued by Li *et al.* (1999). We note that their conclusion is based on the fit result, $a_k = 1.03$, by Titarchuk & Osherovich (1999), who used less accurate expression for viscous frequency (see §2) and ambiguous data for lower kHz QPO frequencies of 4U 1728-34.

3.3. *Correlation between frequencies of low-frequency QPOs and break frequencies in Z sources and atoll sources*

A recent detailed study on the broadband power density spectra of X-ray binaries indicates that the frequency (1-60 Hz) of low-frequency QPOs (including HBOs) have a strong correlation with the break frequency (ν_{break}; 0.1-30 Hz) of both Z sources and atoll sources, possibly as well as black hole candidates (Wijnands & van der Klis 1999). However, the HBO frequencies of Z sources follow a different relationship with ν_{break} from the low-frequency QPOs of other sources. Such a difference has not been satisfactorily explained so far (see, however, Titarchuk *et al.* (1999) for a possible explanation but note that they mistakenly plotted the HBO frequencies of Z sources as the viscous frequencies in their Figure 4).

In TLM, because HBO and viscous frequencies come from different mechanisms, it is natural that they follow different correlations with the break frequencies. According to equation (5) and equations (8)-(10), we can derive the predicted correlation of HBO frequencies, viscous fre-

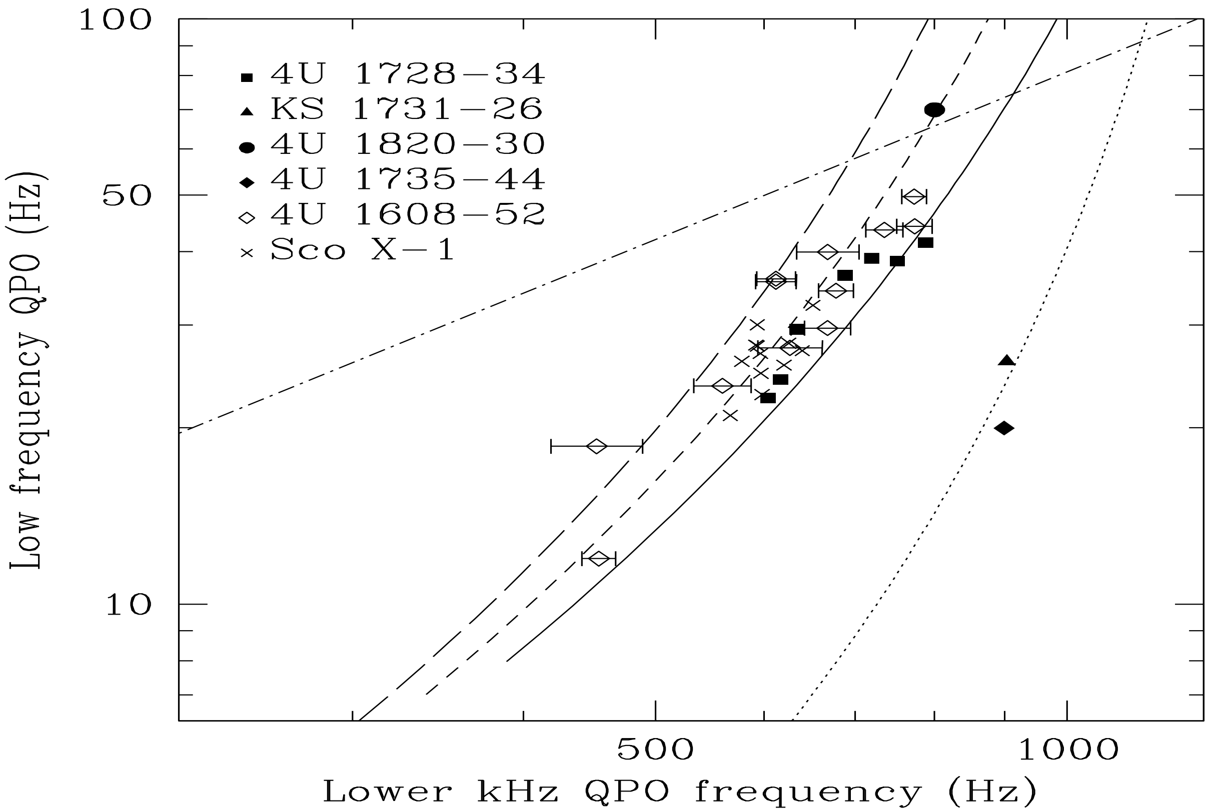

FIG. 3.— The correlation between the frequencies of low-frequency QPOs and lower kHz QPO frequencies. The solid, short dashed and long dashed lines represent the TLM predictions with $\alpha(H/R)^2 = 7.0 \times 10^{-4}$ and $M = 1.2 M_\odot$, $1.4 M_\odot$ and $1.6 M_\odot$, respectively. The dotted line represent the prediction with $\alpha(H/R)^2 = 3.2 \times 10^{-4}$ and $M = 1.4 M_\odot$. The dot-dashed line represents the empirical relation between HBO frequencies and lower kHz QPO frequencies for Z sources.

quencies with break frequencies. In Figure 4, we plot these predictions and compare them with the observation data. For atoll sources, our prediction closely agrees with the relation, $\nu_{break} = 0.04\nu_{LF}^{1.63}$, which was derived from the observed correlations $\nu_{LF} - \nu_{kHz}$ and $\nu_{break} - \nu_{kHz}$ for atoll source 4U 1728-34 (Ford & van der Klis 1998). The viscous frequencies of other atoll sources and Z source Sco X-1 also follow nearly the same $\nu_{LF} - \nu_{break}$ correlation. We adopted $l_d/R_0 = 0.3$ in equation (9) and the same parameters ($\alpha(H/R)^2 = 7.0 \times 10^{-4}$, M=$1.4M_\odot$) as above to derive this correlation. The value, $l_d/R_0 = 0.3$, is appreciated by the observed data and also consistent with our assumption that the diffusion length scale is smaller than the vertical and radial size of the transition layer. Note that $l_d = 0.3R_0 < H$ means H/R may be larger than 0.3. This is quite possible if the magnetic field is low and the radial size of transition layer is small. If we take $\alpha(H/R)^2 = 7.0 \times 10^{-4}$, it requires that α should be less than 0.01. Such a lower value of α is still well within the range of viscosity parameter discussed in the accretion disk model with boundary layer. Assuming $\nu_h - \nu_k = 300$ Hz and δ around 6^o, we derive the correlation between HBO frequencies and break frequencies. Comparing with the observation data of Z sources, we note that the agreement is well. There are several data points for atoll source 4U 1728-34 locating at the branch for HBO in Z sources, which may indicate that these data are actually frequencies of HBO-type QPOs in atoll sources (see §3.2).

3.4. *Variation of peak separation of two kilohertz QPOs*

The observed variation of difference of two kHz QPO frequencies in at least one Z source, Sco X-1 (van der Klis *et al.* 1997) and four atoll sources, 4U 1728-34, 4U1608-52, 4U 1735-44 and 4U 1702-43 (Méndez *et al.* 1998a,b; Ford *et al.* 1998; Méndez & van der Klis 1999; Markwardt *et al.* 1999), seem to exclude any QPO model that predicts the constant peak separation. The magnetospheric beat-frequency model, as well as the sonic-point beat-frequency model, probably can not be responsible for the kHz QPOs, unless further modifications are made.

Recent precise measurements of kHz QPO frequencies show that the peak separation $\Delta\nu = \nu_2 - \nu_1$ decreases considerably when kHz QPO frequencies increase (van der Klis 2000). This feature can not be well fitted by the RPM, which predicts a much steeper decrease of $\Delta\nu$ when ν_2 increases to more than 1000 Hz and a significant decrease of $\Delta\nu$ when ν_2 decreases to less than 700 Hz (Stella & Vietri 1999). The latter effect has not been observed yet. A precise match between RPM and observations requires some

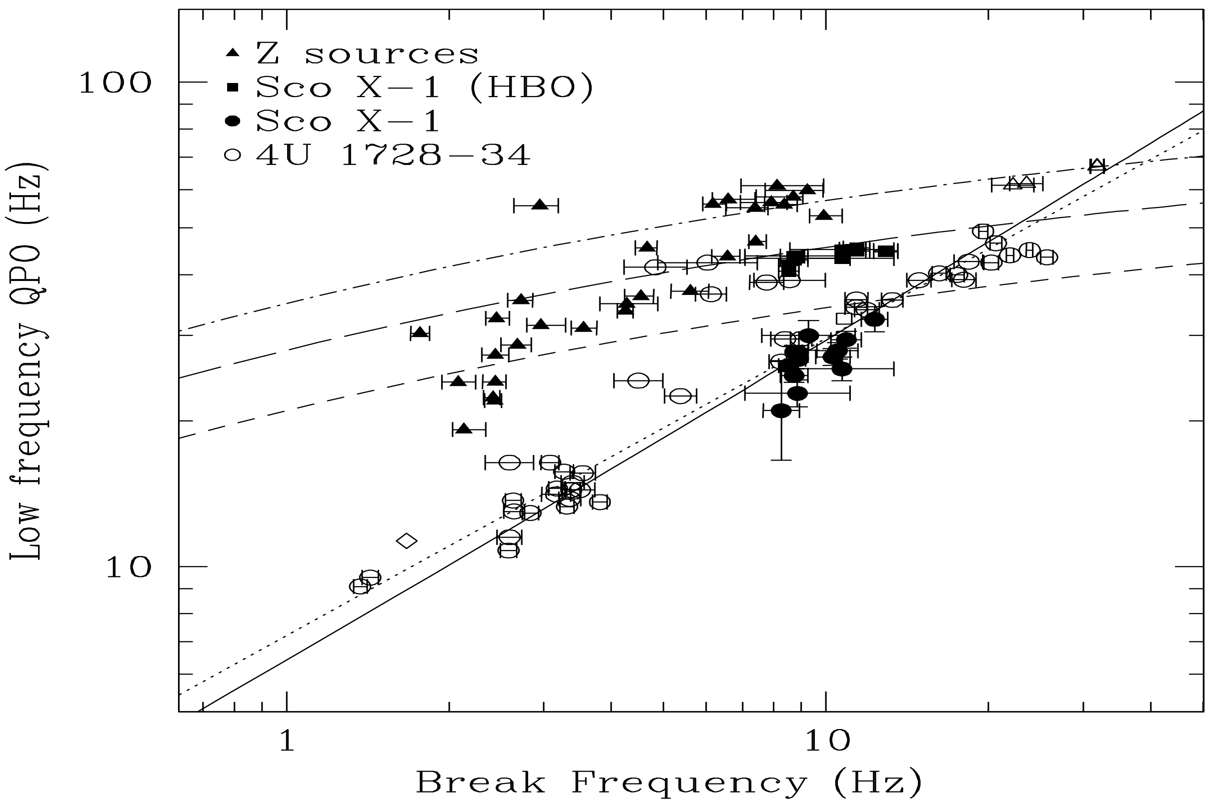

FIG. 4.— The correlation between the frequencies of low-frequency QPOs and break frequencies. The short dashed, long dashed and dot-dashed lines represent the TLM predictions with $\Delta\nu = 300Hz$ and $\delta = 4.5^o$, 6^o, 7.5^o, respectively. The solid line represent the prediction with $\alpha(H/R)^2 = 7.0\times10^{-4}$ and $M = 1.4M_\odot$. $l_d/R_0 = 0.3$ was assumed to derive the theoretical break frequency. The dotted line represents the empirical relation $\nu_{break} = 0.04\nu_{LF}^{1.63}$ derived for 4U 1728-34. Other atoll sources are plotted as open triangles, diamonds and squares.

additional assumptions such as the dependence of orbital eccentricity on the orbital frequency. In TLM, however, the decrease of $\Delta\nu$ as the increase of kHz QPO frequency is a natural result even if we simply take $\Omega = constant$ (see equation (6)). In Figure 5, we show the $\Delta\nu - \nu_1$ relation for all 5 sources with observed $\Delta\nu$ variations. When we plot the TLM prediction assuming the constant $\Omega/2\pi$ (340Hz, 365Hz and 380Hz) in equation (6), we see that even at this simple assumption the theoretical predictions are closely agreement with most observation data of Sco X-1, 4U 1708-52, 4U 1735-44 and 4U 1702-43. However, it is clear that this simple assumption can not explain the data of 4U 1728-34. In fact, the matter in magnetosphere probably experiences differential rotation and Ω may not be a constant. If we assume that the angular velocity could be expressed as $\frac{\Omega}{2\pi} = \nu_0 - (c_1\nu_k^{2/3} - c_2\nu_k^2)^2$ (Osherovich & Titarchuk 1999a) where ν_0, c_1 and c_2 are constants, and take $\nu_0 = 405$ Hz, $c_1 = 0.26\mathrm{Hz}^{-1/6}$ and $c_2 = 3.4\times10^{-5}\mathrm{Hz}^{-3/2}$, we can see the prediction of peak variation of twin kHz QPO frequencies is close to the observation data for 4U 1728-34. In summary, it seems that the variation of peak separation of two kHz QPOs is also consistent with the theoretical prediction of TLM.

4. OBSERVATIONAL DETERMINATION OF MISALIGNMENT OF NEUTRON STAR MAGNETOSPHERE AND ACCRETION DISK

There are only few methods that can be used to estimate the magnetic inclination angle between magnetic axis and neutron star rotation axis for radio pulsars (Lyne & Manchester 1988). It may be more difficult to measure these angles for X-ray pulsars and low-mass X-ray binaries. However, according to TLM (see equation (7)), if we have simultaneous observation data for frequencies of both kHz QPOs and HBOs, we may estimate the angle between magnetophere equator and disk plane for neutron star X-ray binaries. We note that equation (7) is derived under the assumption that δ is small. In fact, a more accurate equation, which is also applicable to arbitrary values of δ, can be directly obtained from the dispersion relation of Keplerian oscillator (see equation (2)), namely

$$\delta = arcsin(\frac{\nu_2\nu_{HBO}}{\nu_1\sqrt{\nu_2^2+\nu_{HBO}^2-\nu_1^2}}), \tag{11}$$

where ν_1, ν_2 and ν_{HBO} are the observed lower, upper kHz QPO and HBO frequencies respectively, and we assume that they correspond to the Keplerian frequency and two characteristic oscillation frequencies derived from the dis-

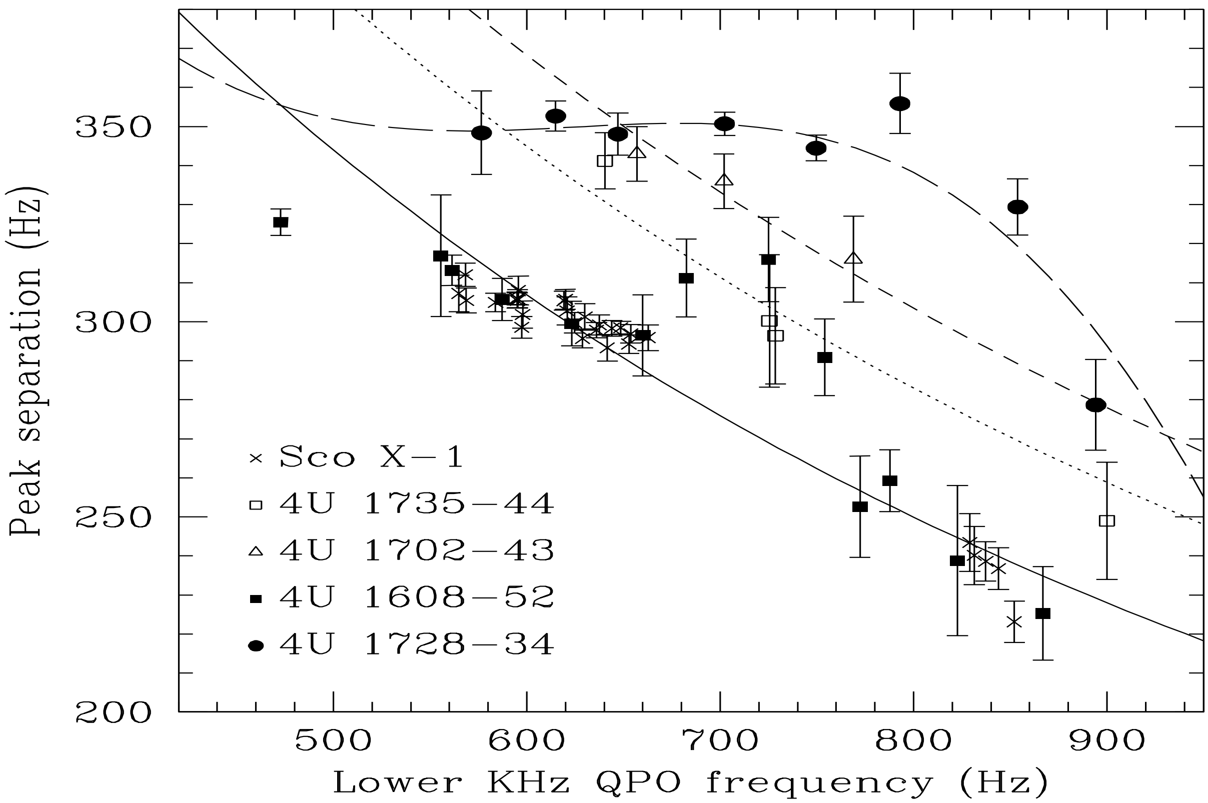

FIG. 5.— The relation between the peak separations of two kHz QPO frequencies and the lower kHz QPO frequencies. The solid, dotted and short dashed lines represent the TLM predictions using equation (6) with $\Omega_0/2\pi$ =340 Hz, 365 Hz and 380 Hz, respectively. The long dashed line represent the prediction using equations (3) and (6) with parameters $\nu_0 = 405$ Hz, $c_1 = 0.26$ Hz$^{-1/6}$ and $c_2 = 3.4 \times 10^{-5}$ Hz$^{-3/2}$.

persion relation (see §2). In the limit $\nu_{HBO} << \nu_2$, equation (11) is identical to equation (7). Note that equation (11) can be also used as a test of TLM because the derived values of δ using different set of simultaneously observed QPO frequency data for each source should be nearly the same (Titarchuk & Osherovich 2000).

Up to know, simultaneous frequency data of both kHz QPOs and HBOs have been obtained with RXTE for five Z sources. In two atoll sources with simultaneous data of two KHz QPO frequencies, the HBO-like frequencies have been identified recently (Titarchuk *et al.* 1999; Osherovich & Titarchuk 1999b). Therefore, the misalignment of magnetosphere and accretion disk can be estimated for these sources according to equation (11). In table 1, we list the observation data and the derived average values of δ for them. It is clear that all of them have smaller δ values and are therefore consistent with the assumption in §2. Especially for Z sources, their δ values are in a very narrow range from $5.^o4$ to $6.^o4$. Moreover, the standard derivations of derived δ values are small, which means nearly the same δ value is obtained for each source using the simultaneously observed QPO frequency data. This is well agreement with the prediction of TLM. In addition, we noted that the method we used to determine δ is better than that used by Osherovich & Titarchuk (1999b) and Titarchuk *et al.* (1999), who derived δ by fitting Ω calculated with the observed kHz QPO frequencies (see equation (2)). It is obviously not necessary to do so if we have simultaneously observed frequency data of HBOs and kHz QPOs. Instead, it is more straightforward and accurate to derive δ by using equation (11). However, their estimated values, $\delta = 3.^o9 \pm 0.^o2$ for 4U 1702-42, $\delta = 8.^o3 \pm 1.^o0$ for 4U 1728-34 and $\delta = 5.^o5 \pm 0.^o5$ for Sco X-1, are very close to our estimations. For other four Z sources in Table 1, it is the first time to give the estimations of their angles between magnetosphere equator and disk plane.

5. DISCUSSION

The comparisons of the model predictions of transition layer model with the observed data seem to suggest that TLM may provide better explanations on them than the current version of other QPO models. In contrary to the inability of beat-frequency model to explain the variation of peak separation of two kHz QPO frequencies and the difficulty of relativistic precession model to explain both the correlations of HBO frequencies with lower and upper kHz QPO frequencies, TLM is so far the only model that can self-consistently explain these observed correlations. In addition, TLM suggests some physics mechanisms for producing low-frequency QPOs and break frequencies in

Table 1
Angle between magnetosphere equator and accretion disk plane determined by simultaneously observed QPO frequency data

Name	Type	ν_2 (Hz)	ν_1 (Hz)	ν_{HBO} (Hz)	Ref.	$< \delta >$ (^{o})
Cyg X-2	Z	856	532	46	1,2	6.34±0.14
GX 17+2	Z	640–1090	480–780	25–61	1,3	6.30±0.37
Sco X-1	Z	870–1080	550–860	41-48	1,4	5.44±0.22
GX 5-1	Z	500–890	210–660	17–51	1,5	5.80±0.62
GX 340+0	Z	560–820	250–500	22-45	1,6	6.40±0.45
4U 1728-34	Atoll	930–1130	600–790	52-90	1,7,8	8.01±1.06
4U 1702-42	Atoll	1000–1080	660-770	33–40	9,10	3.96±0.23

References: (1). Psaltis et al. (1999a); (2). Wijnands et al. 1998a; (3). Wijnands et al. 1997; (4) van der Klis et al. 1997; (5). Wijnands et al. 1998b; (6). Jonker et al. 1998; (7). Strohmayer et al 1996; (8). Ford & van der Klis 1998; (9). Markwardt et al. (1999); (10). Osherovich & Titarchuk (1999b)

X-ray binaries, and can explain the observed correlation between the frequencies of low-frequency QPOs and the break frequencies. On the contrary, no clear mechanisms for these low frequency features have been discussed yet in most of other QPO models.

Another clear difference between TLM and other models is that the lower kHz QPO frequency, rather than the upper kHz QPO frequency, is described as Keplerian frequency. Therefore, it is not necessary in TLM to involve larger neutron star mass and stiff equation of state as required in other models such as sonic-point beat-frequency model (Miller *et al.* 1998) and relativistic precession model (Stella & Vietri 1998, 1999). The compact star in TLM can be still a normal neutron star with mass around $1.4M_\odot$ and radius around 12km. Indeed, the observed correlation of QPO frequencies can be well explained by taking these parameters in TLM. A larger neutron star mass ($> 2M_\odot$) is not appreciated since it will lead to steeper correlation of low-frequency QPO frequencies with lower kHz QPO frequencies than that observed for atoll sources (see Figure 2).

The variations of the size of magnetosphere and the size of transition layer due to the change of mass accretion rate may account for the the observed variations of QPO frequencies in X-ray binaries. Because the size of magnetosphere can be described approximately by the Alvén radius R_A, which is inverse proportional to $\dot{M}^{-2/7}$, the magnetosphere will shrink if accretion rate $\dot{M}$ increases. This may lead to the increases of the kHz QPO frequencies (ν_h, ν_k) and the HBO frequency (ν_L). The size of the transition layer can be described by $R_{out} - R_0$, which is directly related to the Reynolds number of accretion flow (see equation (9)). The increase of accretion rate will cause the increase of Reynolds number, and therefore lead to the decrease of $R_{out}-R_0$. According to equations (8) and (10), both the viscous frequency and the break frequency will increase as the increase of accretion rate. Indeed, many observations have shown the increases of kHz QPO frequencies, HBO frequency and break frequency when the sources move along the tracks in the color-color diagram, equivalently, in the sense of increasing inferred accretion rate (e.g. Méndez & van der Klis 1999). These results are also consistent with the predictions of TLM. Recently, Cui (2000) proposed that the disappearance of kHz QPOs may be explained by the "disengagement" between the magnetosphere and the Keplerian disk. Campana (2000) suggested that the vanishing of the magnetosphere may led to the stopping of the kHz QPO activity. This could be also the natural result of TLM since the Keplerian oscillator has no chance to enter the rotating frame of reference at all if it losts direct contact with the magnetosphere.

We note that the similar QPO frequencies in the range of 1–400 Hz have been observed in X-ray millisecond pulsar SAX J1808.4-3658 (Wijnands & van der Klis 1998a) and Galactic black hole candidates GRO J1655-40 (Remillard *et al.* 1999a), GRS 1915+105 (Morgan, Remillard & Greiner 1997), XTE J1550-564 (Remillard *et al.* 1999b) and XTE J1859+226 (Cui *et al.* 2000). It is still not clear whether the 67–400 Hz QPOs observed in these objects are identical to the kHz QPOs in neutron star LMXBs. But if we assume that these hundred-hertz QPOs are similar to the lower kHz QPOs, the correlations of their frequencies of low-frequency QPOs (1-20Hz) and break frequencies with the frequencies of these 'kHz QPOs' seem to be similar as found for atoll sources (see Psaltis *et al.* 1999a; Wijnands & van der Klis 1999). In fact, if we plot the QPO frequency data for SAX J1808.4-3658 in Figures 3 and 4, these points will locate in the lowerleft parts of both figures but are still consistent with the correlations for atoll sources. Their positions in these two figures also support that SAX J1808.4-3658 is similar to the low-luminosity LMXBs with low accretion rate (Wijnands & van der Klis 1998a). Its estimated magnetic field strength, $B < (2-6) \times 10^8$ Gauss (Wijnands & van der Klis 1998b), is indeed similar as that for atoll sources. For black hole candidates, a similar transition layer as in LMXBs may also exist between a geometrically thin clod disk and a geometrically thick hot disk (or accretion-dominated accretion flow; see Narayan & Yi 1994) due to the sub-Keplerian rotating feature of the inner hot disk. Because of the lack of magnetosphere near the black hole, we can only predict three characteristic frequencies (ν_k, ν_v and ν_{break}) from

TLM. This may help us to understand why we did not observed QPO frequencies larger than 400 Hz for black hole candidates and why their power density spectra and correlations between low and high QPO frequencies are more similar as those for atoll sources than for Z sources (van der Klis 1994, Psaltis *et al.* 1999a). A detailed comparison of these features, however, is beyond the scope of this paper.

We should also mention that as in other QPO models, TLM requires blob be lifted from the inner part of Keplerian disk. It is still unclear which mechanism could lead to such lifting. In addition, most theoretical predictions of TLM are made by assuming the angular velocity of magnetosphere Ω is constant. This may not be the case in reality. Considering a non-constant Ω will lead to the complexity of the dispersion relation and therefore some different characteristic oscillation frequencies. Moreover, the lack of explicit knowledge about the diffusion process in the transition layer may result in the uncertainty in estimating the break frequency. In spite of these questions, TLM should be considered a very competitive model for QPOs in X-ray binaries.

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